Rational Approximations of Irrational Numbers

Student Probe
What is an approximate value of $\sqrt{5}$? How do you know?

Answer: $\sqrt{5} \approx 2.2$. Since $\sqrt{4} = 2$ and $\sqrt{9} = 3$, $\sqrt{5}$ must be between 2 and 3. It is closer to 2 than to 3, because 5 is closer to 4 than to 9.

Lesson Description
This lesson uses benchmark numbers and estimation to help students order and approximate values of common irrational numbers. Only a few irrational numbers are considered. Calculator use is encouraged.

Rationale
As students’ understanding of the real number system deepens and expands, they must make sense of numbers that cannot be expressed as repeating or terminating decimals. These irrational numbers present two concepts that seem paradoxical to students. First, $\sqrt{7}$ (or any irrational number) is an exact value, while 2.64575 is an approximation, no matter how many decimal places it is extended. Secondly, there is a precise point on the number line where $\sqrt{7}$ is located even though it is difficult to locate. Students need to understand the nature of irrational numbers and that the ideas they know about benchmark numbers and approximations with the rational numbers transfer to irrational numbers as well.

Preparation
Prepare copies of Rational Equivalents for each student.

At a Glance
What: Approximate irrational numbers
Common Core Standards: CC.8.NS.2
Know that there are numbers that are not rational, and approximate them by rational numbers. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$). For example, by truncating the decimal expansion of $\sqrt{2}$ (square root of 2), show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Matched Arkansas Standard: AR.8.NO.3.5
(NO.3.8.5) Application of Computation:
Calculate and find approximations of square roots with appropriate technology

Mathematical Practices:
Reason abstractly and quantitatively.
Look for and express regularity in repeated reasoning

Who: Students who cannot approximate irrational numbers.

Grade Level: 8

Prerequisite Vocabulary: square root, irrational number

Delivery Format: small group

Lesson Length: 30 Minutes

Materials, Resources, Technology:
calculator

Student Worksheets: Rational Equivalents
Lesson

<table>
<thead>
<tr>
<th>The teacher says or does...</th>
<th>Expect students to say or do...</th>
<th>If students do not, then the teacher says or does...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. What numbers are called perfect squares? What makes a number a perfect square? We say these perfect squares are rational square roots.</td>
<td>1, 4, 9, 16, 25, ... Some whole number times itself equals the number.</td>
<td>Refer to Factor Pairs.</td>
</tr>
<tr>
<td>2. Compute the rational square roots and record them on your number line. The small tick marks are the location of the rational square roots.</td>
<td></td>
<td>Monitor students.</td>
</tr>
<tr>
<td>3. What about the value of $\sqrt{2}$? Can you estimate its value?</td>
<td>Answers may vary.</td>
<td>Do not correct wrong answers at this time.</td>
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<tr>
<td>4. Let’s see what we know. What is $\sqrt{1}$? What is $\sqrt{4}$? Since 2 is between 1 and 4, $\sqrt{2}$ must be between 1 and 2. Estimate the value of $\sqrt{2}$. Is this value closer to 1, or is this value closer to 2.</td>
<td>1 2</td>
<td></td>
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<tr>
<td>5. Do you think it is closer to 1 or closer to 2? Why?</td>
<td>It is probably closer to 1. Because 2 is closer to 1 than to 4.</td>
<td></td>
</tr>
<tr>
<td>6. Let’s check to see if our theory is correct. Calculate $\sqrt{2}$ with your calculator. Was your estimate correct?</td>
<td>$\sqrt{2} \approx 1.414$ (rounded to the nearest thousandth)</td>
<td>Monitor students as they use the calculator</td>
</tr>
<tr>
<td>The teacher says or does...</td>
<td>Expect students to say or do...</td>
<td>If students do not, then the teacher says or does...</td>
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<td>7. Locate and label the position of $\sqrt{2}$ on your number line.</td>
<td>Correct placement of $\sqrt{2}$.</td>
<td>Monitor students.</td>
</tr>
</tbody>
</table>
| 8. Let’s estimate the value of $\sqrt{3}$.
$\sqrt{3}$ will be between which two whole numbers? How do you know? | It will be between 1 and 2.
$\sqrt{1} = 1$
$\sqrt{3} = \_\_\_\_$
$\sqrt{4} = 2$ | |
| 9. Is this value closer to 1, or is this value closer to 2? How do you know? | It is closer to 2 because 3 is closer to 4 than to 1. | |
| 10. We found that $\sqrt{2} \approx 1.414$. What does this tell us about $\sqrt{3}$? | $\sqrt{3}$ is between 1.414 and 2. | |
| 11. Let’s check to see if our theory is correct. Calculate the value of $\sqrt{3}$ with your calculator. | $\sqrt{3} \approx 1.732$ (rounded to the nearest thousandth) | Monitor and make sure students are using the calculator correctly. |
| 12. Locate and label the position of $\sqrt{3}$ on your number line. | Correct placement of $\sqrt{3}$. | |
| 13. Repeat steps 3-7 with additional irrational numbers on the number line. | | |

**Teacher Notes**

1. Students should understand that the representation of irrational numbers such as $\sqrt{3}$ is precise. The value 1.732 is a rational approximation. The expression $\sqrt{3} \approx 1.732$ should always be written as an approximate value. $\sqrt{3} = 1.732$ is incorrect.

2. $\sqrt{3}$ is a precise point on the real number line, although it is difficult to locate.

3. There are an infinite number of irrational numbers. Some examples include the square root of any non-perfect square, the cube root of any non-perfect cube, etc., $\pi$, $e$, etc.

**Variations**

1. Ask students to estimate large irrational numbers not listed on the handout.
2. Extend the lesson to include other irrational numbers such as $\sqrt{10}$ or $\pi$. 
Formative Assessment

What is an approximate value of $\sqrt{10}$? How do you know?

Answer: $\sqrt{10} \approx 3.2$

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\begin{align*}
\sqrt{9} &= 3 \\
\sqrt{10} &= \boxed{} \\
\sqrt{16} &= 4
\end{align*}
\]

$\sqrt{10}$ must be between 3 and 4, but closer to 3 since 10 is closer to 9 than to 16. $3.1^2 = 9.61$ and $3.2^2 = 10.2$. 3.2 is a good estimate.

References