Properties of Rotations

Student Probe
Find the image of a 50° counterclockwise rotation about point P.

Lesson Description
The lesson examines rotations as the transformation obtained by reflecting an object successively over two intersecting lines, and then modeling rotations in the plane. Students will use patty paper or miras to perform the transformations.

Rationale
By moving all the points of a geometric figure according to certain rules, images of the original figure can be created in a process called transformations. Each point on the original figure corresponds to a point on its image. If the image is congruent to the original figure, the process is called rigid transformation, or isometry. Rotation is a type of isometry in which all the points in the original figure rotate, or turn, an identical number of degrees about a fixed center point. A rotation is defined by its center or point of rotation, the number of degrees it is turned (angle of rotation), and the direction (clockwise or counterclockwise). If no direction is given, assume the direction of rotation is counterclockwise. Using geometric properties of transformations in the coordinate plane allows students to apply transformations to figures and to graphs of functions in the coordinate plane, as well as develop 3-D perspective.

Preparation
Prepare copies of Rotations for each student. Provide students with patty paper, miras, ruler and grid paper.

At a Glance
What: Properties of rotations
Common Core State Standard: CC.9-12.G.CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Matched Arkansas Standard: AR.9-12.CGT.G.5.7 (CGT.5.G.7) Draw and interpret the results of transformations and successive transformations on figures in the coordinate plane:
-- translations,
-- reflections,
-- rotations (90°, 180°, clockwise and counterclockwise about the origin),
-- dilations (scale factor)
Mathematical Practices:
Use appropriate tools strategically.
Look for and express regularity in repeated reasoning.
Who: Students who do not understand rotations
Grade Level: Geometry
Prerequisite Vocabulary: preimage, transformation, rotation, quadrilateral, reflection, coordinates
Prerequisite Skills: perform a reflection, locate points on coordinate plane, identify coordinates of a point.
Delivery Format: Individual or pairs
Lesson Length: 20 min.
Materials, Resources, Technology:
Patty paper, grid paper, ruler, mira
Student Worksheets: Rotations (.pdf)
### Lesson

<table>
<thead>
<tr>
<th>The teacher says or does...</th>
<th>Expect students to say or do...</th>
<th>If students do not, then the teacher says or does...</th>
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<tbody>
<tr>
<td>1. In the diagram, use your patty paper (or mira) to reflect quadrilateral ABCD over the y-axis to obtain $A'B'C'D'$.</td>
<td>Students will use patty paper or mira to reflect the quadrilateral over the y-axis.</td>
<td>Refer to <a href="#">Properties of Reflections</a>.</td>
</tr>
<tr>
<td>2. Reflect $A'B'C'D'$ over the x-axis to obtain $A''B''C''D''$.</td>
<td>Student will use patty paper or mira to reflect the quadrilateral over the x-axis.</td>
<td>Model for students.</td>
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<tr>
<td>3. Find the coordinates of the vertices of $A'B'C'D'$.</td>
<td>A'(4,1), B'(2,3), C'(4,5), D'(5,2.5)</td>
<td>Where are the vertices of the quadrilateral? How do we write the coordinates?</td>
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<tr>
<td>4. Find the coordinates of the vertices of $A''B''C''D''$.</td>
<td>A''(4,-1), B''(2,-3), C''(4,-5), D''(5,-2.5)</td>
<td>Where are the vertices of the quadrilateral? How do we write the coordinates?</td>
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<tr>
<td>5. The result of reflecting the image two times is the same as a rotation. We can find a single rotation that maps ABCD onto $A''B''C''D''$.</td>
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<td>6. Trace ABCD on patty paper. Put a pencil on the point of rotation (the origin) and rotate the original shape onto $A''B''C''D''$.</td>
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<td>Model for students.</td>
</tr>
<tr>
<td>7. How can we describe the rotation?</td>
<td>A rotation of $180^\circ$ around the origin.</td>
<td>Compare the preimage and the resulting image. How much did the image move?</td>
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<td>8. How do the coordinates for the preimage and image compare? Compare the coordinates of the vertices of ABCD and of A”B”C”D”. What do you find?</td>
<td>The coordinates are opposites of each other. If (x,y) are the coordinates of a point of ABCD, then the corresponding point of A”B”C”D” has coordinates (-x, -y).</td>
<td>Compare the coordinates of all 4 vertices of the quadrilateral in the preimage and final image.</td>
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<td>9. Repeat Steps 1-8 with additional figures, if necessary.</td>
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**Teacher Notes**

1. The composition of two reflections about intersecting lines produces a rotation. Its center of rotation is the intersection of the lines. The angle of rotation is equal to twice the measure of the angle formed by the two lines.
2. In order to perform a rotation of an object a point of rotation, angle of rotation and direction (clockwise or counterclockwise) need to be established.

**Variations**

Software such as Geometer’s Sketchpad or Geogebra may be used.

**Formative Assessment**

Find the image of a 100° clockwise rotation around point P.
References
1. The two triangles are equivalent by a rotation. Where is the center of this rotation?

2. A rotation sends figure A to figure B. Draw where this rotation sends the yellow square.