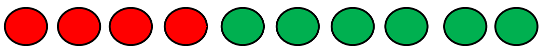
Part to Part Relationships

Student Probe

Jerry has a set of 10 marbles pictured below. He needs some help describing the amount of marbles he has in his collection. Use the picture below to help Jerry answer questions A, B, C, and D.



A) Write a fraction describing the relationship between the red and green marbles in the set.

B) Write a fraction describing the relationship between the red marbles and the entire set.

C) Write a fraction describing the relationship between the green marbles and the entire set.

D) Jerry's friend Maria tells him that for every 2 red marbles there are 3 green marbles. Is she correct? Why or why not?

Answers

A) 4:6, 4 to 6, 4 for every 6, 4/6
Note: Watch for students who give the fractional amount for the red or green marbles. This suggests that they are only looking at a part to whole relationship and need to work further with this lesson on part to part relationships.

B) Red 4:10, 4 out of 10, 4/10

C) Green 6:10, 6 out of 10, 6/10
Note: If students cannot answer B or C correctly they need additional work on the part to whole relationship and naming conventions for fractions.

D) Yes, Maria is correct. Students' explanations should contain information related to the idea of "ratio", although the term is not expected to be used the concept is central to this lesson topic.
Note: If students can answer part A correctly but cannot explain why Maria is correct then continued work on part to part relationships is required.

At a Glance

What: Understanding part to part relationships with fractions

Common Core Standards: CC.4.NF.1 Extend understanding of fraction equivalence and ordering. Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

Matched Arkansas Framework: AR.5.NO.1.1 (NO.1.5.1) Rational Numbers: Use models and visual representations to develop the concepts of the following:
---Fractions: parts of unit wholes, parts of a collection, locations on number lines, locations on ruler (benchmark fractions), divisions of whole numbers;
---Ratios: part-to-part (2 boys to 3 girls), part-to-whole (2 boys to 5 people);
---Percents: part-to-100

Mathematical Practices:
Make sense of problems and persevere in solving them.

Who: Students who do not understand part to part relationships

Grade Level: 4

Prerequisite Vocabulary: numerator, denominator, part to whole

Prerequisite Skills: naming fractions, part to whole relationships,

Delivery Format: individual, small group

Lesson Length: 15-30 minutes

Materials, Resources, Technology:
Red/Yellow color counters

Student Worksheets: Part to Part Relationships: Using Colored Counters

Lesson Description

The lesson is intended to help students develop an understanding of the existence of relationships other than “part to whole”. Students will be given repeated exposure to physical models and repeated questioning about how one colored piece relates to another rather than to the whole. It is through these guided experiences that students will be able to generalize the situation and conceptualize the part to part (ratio) relationship.

Rationale

Students often fail to understand that fractions can be used to express relationships other than part to whole. Most experiences students receive with fractions involve part to whole comparisons. If repeated exposures and opportunities to explore fraction concepts based on part to part associations are not given, students do not get a solid foundation for future work with ratio and proportion.

Preparation

Provide students with red and yellow color counters. Prepare copies of Part to Part Relationships: Using Colored Counters for each student.

Lesson

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. Take out 1 yellow color counter and 1 red color counter. What fraction of the color counters is red? What fraction of the color counters is yellow? How would you describe the relationship between the yellow and the red counters?	½ of the color counters are red. ½ of the color counters are yellow. There is the same amount of red and yellow counters.	Teacher may need to revisit naming conventions and point out that in the set model 1 out of 2 counters are red and 1 out of 2 counters are yellow. If students are struggling with naming fractions using a set model, then a prerequisite lesson is required before continuing on with part to part ratios.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
2. In order to keep the relationship between these two parts the same what would need to be done if another yellow counter was to be added to the group? What if 3 yellow counters were now placed in the group? Why?	If two yellow counters are in the group that would require two red counters to be in the group in order for the relationship (ratio) to stay the same. If three yellow counters are placed in the group, then three red are required. There would not be the same amount of each.	Place emphasis on keeping the “ratio” the same between the red and yellow (1 to 1). Refer to physical model if students are looking at the whole instead of to the two parts.
3. What would need to be done to keep the relationship between the two parts the same if 100 yellow counters were now placed in the set of color counters? Why?	There would need to be 100 red color counters because that is the only way the ration between the two parts would stay the same.	
4. This type of relationship between numbers is different than a part to whole relationship. We are now comparing one part of a set to another part of the same set. As long as the relationship between the two amounts stays intact the two parts will always have the same amount.		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
5. Take out 1 yellow color counter and 2 red color counters. What fraction of the color counters is red? What fraction of the color counters is yellow? In order to keep the relationship between these two parts the same, what would need to be done if one more yellow counter was to be added to the group?	2/3 of the counters are red. 1/3 of the counters are yellow. If there were 2 yellow counters in the set, that would require 4 total red counters in the set to keep the ratio the same (2 red for every 1 yellow)	
6. How would you describe the relationship between the yellow and the red counters? Why would I need to place 6 red counters in the group?	There is twice as many red color counters as yellow counters in this group. With 3 yellow counters in the group, there would need to be 6 red counters. In order to keep twice as many red as yellow in the set of counters.	
7. In order to keep the relationship the same, 2 red counters must be matched up for every 1 yellow counter. So the amount of red counters is always "how many times bigger than yellow"?	Red counters are always twice as many as yellow counters.	Place emphasis on the *part to part comparison*; instead of part to whole.
8. What would need to be done if 100 yellow counters were now included in the set of counters?	100 yellow to 200 red	

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
9. Is there an easy way we can numerically describe this relationship between the red and yellow counters?	If I want to know about red counters in terms of yellow counters, then I would say that the relationship is 2 reds for every 1 yellow or 2 to 1, 2:1, or 2/1 If I want to know about yellow counters in terms of red counters, then I would say that the relationship is 1 yellow for every 2 reds or 1 to 2, 1:2, or 1/2.	
10. Take out 1 yellow color counter and 3 red color counters. What fraction of the color counters is red? What fraction of the color counters is yellow? In order to keep the relationship between these two parts the same what would need to be done if another yellow counter was to be added to the group?	¾ of the counters are red. ¼ of the counters are yellow. 2 yellow counters would need 6 red counters to keep the "ratio" the same.	
11. How would you describe the relationship between the yellow and the red counters? What if 3 yellow counters were now placed in the group? Why would I need to place 9 red counters in the group?	It appears as though the red counters are always 3 times as many as the yellow. That would require 9 red counters. That would keep the red amount always 3 times as many as the yellow.	

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
12. In order to keep the relationship the same, 3 red counters must be matched up for every 1 yellow counter. So the amount of red counters is always how many times bigger than yellow?	There are 3 times as many red counters as yellow counters.	
13. Is there an easy way we can describe this relationship between the red and yellow counters?	If I want to know about red counters in terms of yellow counters, then I would say that the relationship is 3 reds for every 1 yellow or 3 to 1, 3:1, or 3/1 If I want to know about yellow counters in terms of red counters, then I would say that the relationship is 1 yellow for every 3 reds or 1 to 3, 1:3, or 1/3.	
14. Take out 2 red color counter and 3 yellow color counters. What fraction is red? Yellow? This combination gives me a relationship of 2 red counters for every 3 yellow counters. How can I write a fraction that describes how the red counters relate to the yellow counters?	2/5 is red. (part to whole) 3/5 is yellow. (part to whole) 2/3; 2 red for every 3 yellow (part to part)	Keep asking students to compare part to part and NOT part to whole. Call attention to the fact that the "ratio" must remain the same. Use students understanding of equivalent fractions to help make connections. (Equivalent fraction concepts are not a requirement to see part to part relationships.)

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
15. Use the student worksheet Part to Part Relationships: Using Colored Counters to list the appropriate information about the fractional relationships between the red and yellow counters. (Note: All fractional values in the table will be equivalent fractions.)		
16. In order to keep the relationship between these two parts the same we need to continue to add groups of 2 red and 3 yellow. Let's add another identical group (a group of 2 red and 3 yellow) of red and yellow counters. Use the lab sheet to fill in the information for each color.		The teacher may need to repeatedly pull out and focus on identical groups and how that impacts each new fractional value entered into the table.
17. What fraction name can we give to the new set of red and yellow counters?	4/6: 4 red for every 6 yellow counters	

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
18. What will our new set of colored counters look like if we continue to add another identical group? Use the colored counters to create our new set. What fraction name can we give the new set of counters? Why can't we just add 1 red color counter instead of always adding 2 red each time and 3 yellow?	This will give us two more red making 6 and three more yellow making 9. 6/9: 6 red for every 9 yellow counters You can't just add one red because that would not keep the relationship intact.	Note: If I wanted to take half of 2 red in order to get 1, then I would need to take half of 3 getting 1 ½. The fraction would then be a complex fraction 1/1 ½. Students are probably not ready to try and deal with this concept at this point.
19. The process is repeated for the last set of numbers on the lab sheet.	The last row on the lab sheet would produce 8 red counters for every 12 yellow counters.	
20. Draw a picture that has a total of 15 counters (red and yellow combined) that shows a relationship where for every 2 red counters there are 3 yellow counters.		Note: This is identical to the problem they just completed working. Teacher should pay particular attention to see if students connect the picture they are asked to draw with the physical model they just made with the color counter.

Teacher Notes

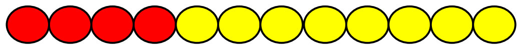
None

Variations

None

Formative Assessment

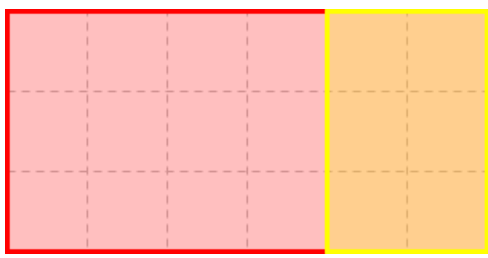
Use the Pictures below to answer questions A, B and C.



A) Write a fraction describing the relationship between the red and yellow counters in the set.

B) Write a fraction describing the relationship between the red and the total counters in the set.

C) Is this statement correct: "For every 3 red counters there are 6 yellow counters."? Explain why or why not.



D) Write a fraction describing the relationship between the yellow shaded area and the red shaded area. Explain how you know your answer is correct.

References

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