Distance Using the Pythagorean Theorem

Student Probe
What is the length of \( \overline{AB} \)?

The length of \( \overline{AB} \) is \( \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = \sqrt{9 \cdot 5} = 3 \sqrt{5} \approx 6.708 \).

Students may incorrectly answer 6 because they count the number of squares through which the segment passes.

Lesson Description
This lesson teaches students to find the length of a line segment using the Pythagorean Theorem directly rather than using the standard distance formula. Students are asked to create a right triangle using the given segment as the hypotenuse. Students are expected to have access to calculators with a square root function.

Rationale
When asked to determine the length of a line segment in the coordinate plane, students are usually expected to use the standard distance formula \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \). However, students frequently fail to use this formula correctly. Since this formula is derived from the Pythagorean Theorem, it is always possible to use the theorem rather than the formula.

Preparation
Create a visual display for graphing the points you will work with the students.

At a Glance
What: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system
Common Core State Standard: CC.8.G.8
Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Mathematical Practices:
Reason abstractly and quantitatively.
Model with mathematics.
Use appropriate tools strategically.
Who: Students who cannot determine the length of a segment in a coordinate plane
Grade Level: 8
Prerequisite Vocabulary: horizontal, vertical, right triangle, coordinates, Pythagorean Theorem, hypotenuse
Prerequisite Skills: Find distance on a number line, apply Pythagorean Theorem to right triangles, graphing in a coordinate system
Delivery Format: small group
Lesson Length: 15-30 minutes
Materials, Resources, Technology: calculator with square root function, grid paper, and straight edge for each student
Student Worksheets: None
### Lesson

<table>
<thead>
<tr>
<th>The teacher says or does...</th>
<th>Expect students to say or do...</th>
<th>If students do not, then the teacher says or does...</th>
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<tbody>
<tr>
<td><strong>1.</strong> Draw an x-and y-axis on your grid paper.</td>
<td>Correctly draw and label an axis system.</td>
<td>Refer to lesson <a href="#">Graph Ordered Pairs on a Coordinate Plane</a>.</td>
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<tr>
<td><strong>2.</strong> Plot A 1,2 and C 5,2. Label your points.</td>
<td>Correctly plot and label points A and C.</td>
<td>Refer to lesson <a href="#">Graph Ordered Pairs on a Coordinate Plane</a>.</td>
</tr>
<tr>
<td><strong>3.</strong> What is the length of ( \overline{AC} )? Write your answer on your drawing.</td>
<td>4</td>
<td>Let’s count the units from A to C.</td>
</tr>
<tr>
<td><strong>4.</strong> Plot B 5,7. Label point B. Draw ( \overline{BC} ).</td>
<td>Correctly plot and label the point and the segment.</td>
<td>Refer to lesson <a href="#">Graph Ordered Pairs on a Coordinate Plane</a>.</td>
</tr>
<tr>
<td><strong>5.</strong> What is the length of ( \overline{BC} )? Write your answer on your drawing.</td>
<td>3</td>
<td>Let’s count the units from C to B.</td>
</tr>
<tr>
<td><strong>6.</strong> What kind of triangle is ( \triangle ABC )? How do you know?</td>
<td>It is a right triangle, because ( \angle C ) is a right angle.</td>
<td>Do you remember the Pythagorean Theorem? Can someone tell us what the Pythagorean Theorem says?</td>
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<tr>
<td><strong>7.</strong> What is true about all right triangles?</td>
<td>Many answers are possible, but listen for ( a^2 + b^2 = c^2 ).</td>
<td>Do you remember the Pythagorean Theorem? Can someone tell us what the Pythagorean Theorem says?</td>
</tr>
<tr>
<td><strong>8.</strong> What is the name we give to ( \overline{AB} )?</td>
<td>It is the hypotenuse of the triangle.</td>
<td>Refer to lesson <a href="#">Pythagorean Theorem and Right Triangles</a>.</td>
</tr>
<tr>
<td><strong>9.</strong> What is the length of ( \overline{AB} )? Explain how you know.</td>
<td>5, because ( 4^2 + 3^2 = 16 + 9 = 25 ) [ \sqrt{25} = 5 ]</td>
<td>Refer to lesson <a href="#">Pythagorean Theorem and Right Triangles</a>.</td>
</tr>
<tr>
<td><strong>10.</strong> Now let’s plot ( D -3,2 ) and ( E 5,1 ). Draw ( \overline{DE} ).</td>
<td>Correctly plot and label the points and the segment.</td>
<td>Refer to lesson <a href="#">Graph Ordered Pairs on a Coordinate Plane</a>.</td>
</tr>
<tr>
<td><strong>11.</strong> How can we find the length of ( \overline{DE} )? Draw a right triangle with hypotenuse ( \overline{DE} ).</td>
<td>Make a right triangle with hypotenuse ( \overline{DE} ).</td>
<td>Model the right triangle.</td>
</tr>
<tr>
<td><strong>12.</strong> Use your ruler to draw a vertical line through D. Now use your ruler to draw a horizontal line through E.</td>
<td>Draw the intersecting lines.</td>
<td>Model how to draw the right triangle.</td>
</tr>
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<td>13. What are the coordinates of the intersection of the horizontal and vertical lines that you just drew?</td>
<td>$-3,1$</td>
<td>Refer to lesson <a href="#">Graph Ordered Pairs on a Coordinate Plane</a>.</td>
</tr>
<tr>
<td>14. Let’s call this point F. It is the vertex of a right angle. What kind of triangle is $\Delta DEF$?</td>
<td>$\Delta DEF$ is a right triangle.</td>
<td></td>
</tr>
<tr>
<td>15. What is the length of $DF$? What is the length of $EF$?</td>
<td>1, 8</td>
<td>Let’s count the units from D to F. Let’s count the units from E to F.</td>
</tr>
</tbody>
</table>
| 16. Let’s use the Pythagorean Theorem to find the length of $DE$. | $8^2 + 1^2 = 64 + 1 = 65$  
$\sqrt{65} \approx 8.062$ | Refer to lesson [Pythagorean Theorem and Right Triangles](#). |
| 17. Now you try one. Let $A\ 9,-7$ and $B\ 0,-2$. Find the length of $AB$. (Monitor students as they work on the problem. If they become confused, prompt them to follow the steps just discussed.) | C should have coordinates $9,-2$ or $0,-7$.  
Length of $AC$ is 9.  
Length of $BC$ is 5.  
$9^2 + 5^2 = 81 + 25 = 106$  
$\sqrt{106} \approx 10.296$ | |
| 18. Let’s try two new points: $E\ -6,6$ and $F\ 9,-5$. Look back at our work in the previous problems. Can you use the same type of calculations to determine the distance between E and F without drawing any pictures? (Monitor students as they work on the problem. If they become confused, prompt them to follow the steps just discussed.) | $11^2 + 15^2 = 121 + 225 = 346$  
$\sqrt{346} \approx 18.601$ | If G is the vertex of the right angle, G will have coordinates $-6,-5$ or $9,6$.  
Length of $EG$ is 11.  
Length of $FG$ is 15. |
Teacher Notes:
1. There are two possibilities for the right angle vertex for each segment. In general, for segment endpoints \( x_1, y_1 \) and \( x_2, y_2 \), the possible vertices will be either \( x_2, y_1 \) or \( x_1, y_2 \). The correct length can be found using either vertex.
2. Students may have difficulty finding the right angle vertex. Grid paper will help students in locating the point and in finding the length of the resulting segments. This step may need to be extensively modeled.
3. In this lesson, irrational answers have been rounded to the nearest thousandth.

Variations
None

Formative Assessment
Let \( A \ 5,-10 \) and \( B \ -3,10 \). Find the length of \( \overline{AB} \).

Answer:
C should have coordinates \( 5,10 \) or \( -3,-10 \).

Length of \( \overline{AC} \) is 8.
Length of \( \overline{BC} \) is 20.
\[ 8^2 + 20^2 = 64 + 400 = 464 \]
\[ \sqrt{464} \approx 21.541 \]

References