Composing and Decomposing Two-dimensional Figures

Student Probe

Give the student two congruent right triangles and ask the student to create new figures. Ask the student to explain the new figures. If the student appears to have never composed figures to make new figures or if the student does not know the geometric language to discuss the figures, this lesson will address this deficit.

Lesson Description

Students will cut their own tangram puzzle to explore the relationships of 2-dimensional figures, specifically, quadrilaterals (square, rectangle, trapezoid, and rhombus) and similar right triangles. Students will use geometric and fractional language as they compose and decompose figures using the tangram pieces. This is a two-part lesson.

Rationale

Students need many hands-on experiences to develop concepts of shape, size, symmetry, congruence, and similarity within both two- and three-dimensional figures. These experiences, along with much student discussion, will help students develop the appropriate vocabulary to build understanding of geometric properties and relationships. In middle school, more formal generalizations will be addressed.

Preparation

Each student will need a 6 inch square of construction paper and scissors. The teacher will also need a square to use to model the folds and cuts. Have several squares available so students can make mistakes and begin again as needed.

In Part 2 of the lesson, outlines of the new figures created with three triangles can be created to scaffold the learning for some students. See this part of the lesson for more details.

At a Glance

What: Discuss attributes of figures that have been composed and decomposed

Common Core State Standard: CC.2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Mathematical Practices:
Make sense of problems and persevere in solving them.
Look for and make use of structure.
Attend to precision.

Who: Students who need to learn how shapes compose and decompose and the corresponding vocabulary to discuss the attributes of the figures.

Grade Level: 2

Prerequisite Vocabulary: Helpful to already know the language but not a requirement - triangle, square, rectangle, rhombus, trapezoid

Prerequisite Skills: Helpful to have worked with pattern blocks

Delivery Format: Individual or small group

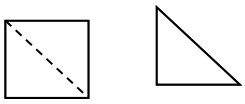
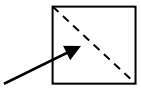
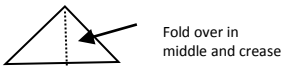
Lesson Length: 20 to 30 min.

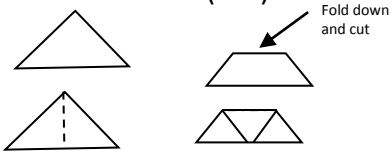
Materials, Resources, Technology: 6- inch squares of construction paper, outlines of composed figures for part 2, pattern blocks (square, trapezoid, rhombus), various examples of rectangles, square, triangles, trapezoids, parallelograms

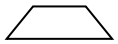
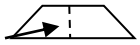
Student Worksheets: None

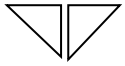
Lesson

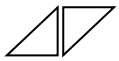
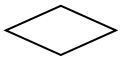
The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
1. In front of you is a 6 inch square piece of paper. What do you know about squares? How do you know whether or not this is a square?	There are four sides and four corners. All sides are equal All corners (angles) are equal.	Show the student a triangle. Is this a square? How do you know? Show a rectangle. Is this a square? How do you know? Then ask again, so how do you know this is a square?
2. How can we prove that all sides have the same length or equal lengths?	Measure the sides.	How can we find out if this side (point) is the same length as all other sides (point)?
3. How could we measure? I don't have a ruler but I do have these straw stirrers that I cut and they are each one inch long. Let's use these to measure the sides to see if they are the same length. (Have coffee stirrers cut in one inch lengths available. You will need at least 12 so students can compare the two sides and see their equivalence more easily.)	With a ruler, student begins lining up the straws next to the sides. (Make sure students line up the straws correctly and that there are no spaces between each segment (iteration). Each side should prove to be 6 straw pieces long.)	If a student has a different way to measure, go with it. Possibilities: Folding the paper over to see if the sides line up Using some other non-standard unit to measure.
4. Now we are going to fold and cut the square into a tangram puzzle. Then we are going to make shapes with the pieces. Who has ever worked with tangrams?	Yes When and what did you do with the tangrams? No	I will do it with you. First I'll show you with my square and then you can copy me. Ready?
5. First we are going to fold the square in half diagonally. Who knows what diagonally means?	The student shows with his hand or arm a diagonal line.	Show the student a diagonal line. Write the word and make an arrow to show what it means. Diagonal 

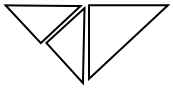
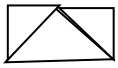
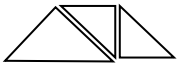
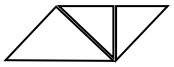
The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
6. Fold it in half like this. (Demonstrate the diagonal fold.) Make sure your corners are touching before creasing. Fold over. 	Student folds and creases.	Student folds but does so irregularly. Help the student fold more accurately.
7. Now cut on the diagonal line. 	Student cuts on the diagonal.	Let the student cut. The student may need to use more than one square if necessary.
8. What happened? What did we make when we cut the square in half on the diagonal?	We made two triangles.	Hold up one of the triangles. What is this?
9. Mathematicians call this a "right" triangle because one of the corners is like one corner of a square. Which corner or angle has a corner or angle like a square? How can we use one of our other squares to prove the triangle has a "square" corner or a right angle?	Student points to the right angle. Student lays the triangle over the square and lines up the right angles.	Take a square and have the student match the right angle to the square angle or right angle of one of the triangles.
10. Now let's take one of these triangles. Fold it in half and cut on the fold. Demonstrate the fold and cut. Remind students to line up the corners before folding.  	Student watches; then folds and cuts.	Show again. Student may need to do this more than once.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
11. Now cut on the crease. (Demonstrate)	Student cuts on the crease.	Student needs to watch again or, if necessary, gets a new piece to fold and cut.
12. What did we create when we folded the right triangle in half and cut it? How do you know?	2 right triangles.  Make the student prove they are right triangles.	2 triangles What type of triangle?
13. Now we are going to use the other large triangle - the triangle that is ½ of the original square. Which one is that? How could we prove it is one half of the original square?	Student finds the large triangle. Student points to the large triangle. To prove it is one half of the original, the student should put all pieces back together to form the square.	Help the student put the pieces back together to see that the large triangle is ½ of the original square.
14. With the other large right triangle, we are going to make 2 folds. Pay close attention. This is trickier. First we are going to fold the right triangle in half. Then we are going to open it back up. Now watch as I fold the top corner down so the tip is touching the base or bottom of the triangle. I am going to cut on the fold. (Cut)  Cut-off the top corner folded down.	Student watches. Student follows your demonstration. Have other large right triangles cut so you can demonstrate a second or third time if needed.	Student needs help with folding and cutting.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
15. Now what shapes did we make? Where have we seen this before?	A trapezoid or quadrilateral and a triangle. Pattern blocks Have pattern blocks available to compare.	You have a trapezoid or a quadrilateral and a triangle. Write the words and put a picture with it. Trapezoid  (quadrilateral)
16. Trapezoids and squares are both quadrilaterals. A triangle is not. What do trapezoids and squares have in common?	They both have 4 sides. Say: Quad means 4.	Put the figures next to each other to see what is the same about both. Why is a triangle not a quadrilateral?
17. Now fold the trapezoid in half like this and cut on the fold. Cut  Now what do we have?	Student cuts in half. 2 smaller trapezoids or quadrilaterals.	Help student fold and cut if necessary.
18. Now fold one smaller trapezoid to make a square and a triangle. Cut.  cut	Demonstrate for student and then have the student try.	Adjust the fold for the student if necessary before cutting.
19. Fold the last trapezoid to make a parallelogram and a triangle. Cut.  Cut	Show student and then have them try. Adjust the fold if necessary before cutting.	
20. Now we are going to see what figures you can make with just the two smaller triangles.		

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
21. Let's see if you can make a square with the two smaller triangles. Let's trace around the pieces.	 Student traces around the pieces.	Keep rotating the pieces around. Don't give up. Help the student hold the pieces to trace.
22. What figure is one half of the square?	A right triangle.	Point to ½. What shape is that?
23. Now try to make a right triangle with the two smaller right triangles. Can you?		Keep rotating the pieces around. Don't give up.
24. What shape is one half of the triangle?	A right triangle	Point to ½. What shape is that?
25. Now trace around the pieces to show that 2 right triangles can also make another larger right triangle.	Student traces the new figure.	Help the student trace.
26. Have the students compare the square they made in Step 25 to the larger right triangle they made in Step 23. What do you notice?	They are the same two pieces; just moved around.	Help students notice the comparisons.

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
27. Now make a rhombus with the two smaller right triangles. (Show the shape with the pattern block and write this word)		Keep rotating the pieces around. Don't give up.  rhombus
28. Now trace it.	Student traces.	Student needs help tracing.
29. What shapes did we make with two triangles? Let's rotate the shapes to make all three shapes again. What stays the same? What is ½ of each new figure? What changes?	A square A larger right triangle A rhombus Same: Each new figures is made of 2 right triangles. ½ of the new figure is one right triangle. Different: Sides are combined to make different figures.	Help student to rotate shapes and notice what is the same and what is different.
PART 2 (This part of lesson should be implemented on a different day.)		
30. Have students use the two right triangles to recreate the square, the larger right triangle, and the rhombus as a review. Have them discuss what is the same and what is different about the construction of the figures.		
31. Today we are going to use three right triangles to make a square, a larger right triangle, a rectangle, and a trapezoid. Do you think we can?	Yes	Not sure. Say: I'll help you.
32. Now I want you to take the same two small triangles we used last time and also one medium triangle. Try to make a square.		Have a square outline available for the student to use to fill in with the pieces (must be prepared ahead of time)

The teacher says or does…	Expect students to say or do…	If students do not, then the teacher says or does…
33. Now trace your new figure.	Student traces figure.	Help student trace by holding pieces.
34. Now try to make a triangle with three small triangles.		Have a triangle outline available. (prepared ahead)
35. Now trace your new figure.	Student traces figure.	Help student trace by holding pieces.
36. Now make a rectangle with the same pieces.		Have a rectangle outline available.
37. Now trace your new figure.	Student traces figure.	Help student trace by holding pieces.
38. Now make a trapezoid with the same pieces.		Have a trapezoid outline available.
39. Now trace your new figure.	Student traces figure.	Help student trace by holding pieces.
40. Now make a rhombus with the same pieces.		Have a rhombus outline available.
41. Now trace your new figure.	Student traces figure.	Help student trace by holding pieces.
42. Have students make comparisons. What is the same? What is different?		

Teacher Notes

1. It is important for the students to cut their own tangram pieces so they can see and believe that the square is made up of all the smaller pieces.
2. Be sure you have practiced this ahead of time so you feel confident when making folds and cuts.
3. It is important to have many spare pieces on hand to use even though students will be making their own. If student pieces are too irregular, you may need to give students your pieces to use to make the new figures.
4. It is good practice for students to fold and cut to make tangram pieces. This could be set in a workstation or a center for students to do many times until they are able to do it easily. This can take much practice.
5. In Part 2, have outlines available of the new figures created – you will need to trace and make these before the lesson. Do not give this outline to the student unless the student becomes frustrated. The outline serves as a puzzle structure to be filled in with the parts.
6. Use precise geometric language when discussing figures and parts of figures. Whenever a new word emerges, write the word on a sentence strip and make a picture next to it to help the student remember the word.

Variations

The Greedy Triangle, by Marilyn Burns (1994), is a great literature source to use with this lesson. Students make figures using triangles.

Extensions

1. Have the students use the 5 smaller pieces (all but the 2 large triangles) to make the same figures – square, right triangle, rectangle, trapezoid, and rhombus.
2. Have the students use all 7 pieces to make all the figures.

Formative Assessment

Give students two triangles and ask them to make all the different figures they can. Ask them to discuss the new figures. What is the same? What is different? Have the student draw a picture incorporating the shapes.

References

About Teaching Mathematics, A K-8 Resource , by Marilyn Burns, 1992, p. 83.