Addition of Fractions

**Student Probe**
What is \( \frac{1}{3} + \frac{2}{5} \)?

A common student misconception is \( \frac{1}{3} + \frac{2}{5} = \frac{3}{8} \) since 1+2=3 and 3+5=8.

**Lesson Description**
This lesson is intended to help students develop an understanding of addition of fractions with unlike denominators followed by the introduction of an algorithm for efficiency. All addends and sums in this lesson are less than one. At the end of the lesson students will be expected to make a sketch of the problem situation and solve it using an algorithm.

**Rationale**
The addition of fractions, like addition of whole numbers, is counting the number of items in the union of two or more sets. However, students frequently fail to understand that the items being added must be alike. Just as the sum of 3 boys and 2 girls is 5 children, the sum of fifths and thirds is expressed as fifteenths. After this is understood, students will be able to employ an algorithm to perform the operation more efficiently. When they successfully master this skill, students will be able to move easily into subtraction of proper fractions and addition and subtraction of mixed numbers.

**Preparation**
Create a visual display for Max’s Brownies and the other problems that you will work with the class.

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**At a Glance**

What: Addition of fractions (less than 1) with unlike denominators


Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

Mathematical Practices:
Make sense of problems and persevere in solving them.

Who: Students who cannot add fractions (less than 1) with unlike denominators

Grade Level: 5

Prerequisite Vocabulary: numerator, denominator, least common multiple, equivalent fractions

Prerequisite Skills: Students can find the least common multiple of two numbers, find equivalent fractions, and add fractions with like denominators

Delivery Format: Individual, small group

Lesson Length: 30 minutes to 1 hour

Materials, Resources, Technology: Visual display for problems such as white board, chalk board, chart paper, etc. Cuisenaire Rods (optional)

Student Worksheets: [Combining Brownies](.pdf)
### Lesson

<table>
<thead>
<tr>
<th>The teacher says or does...</th>
<th>Expect students to say or do...</th>
<th>If students do not, then the teacher says or does...</th>
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<tbody>
<tr>
<td>1. Max made two pans of brownies for a party. The pans were the same size. In one pan Max cut the brownies into 3 pieces of the same size. In the other pan Max cut them into 5 pieces of the same size. On your worksheet, draw how the brownies were cut.</td>
<td>Verify that the students divided one rectangle into 3 congruent rectangles and the second rectangle into 5 congruent rectangles.</td>
<td>Model the correct drawing with the students. Ask “How many pieces do we need in the first pan? How many pieces do we need in the second pan?” Ask the student(s) to count the number of pieces aloud.</td>
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<td>2. After the party, there was 1 brownie left in the pan that had been cut into 3 pieces and there were 2 brownies left in the pan that had been cut into 5 pieces. On your worksheet shade in the brownies that were left.</td>
<td>Verify that the students shaded 1 brownie in one pan and 2 in the other pan.</td>
<td>Model the correct drawing with the students. Ask “How many pieces are left in the first pan? How many pieces are left in the second pan?” Ask the student(s) to count the number of pieces left aloud. If students are unable to name the fractions, refer to the lesson Naming Fractions.</td>
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<tr>
<td>3. What part of the brownies was left in the first pan?</td>
<td>Expect to hear ( \frac{1}{3} ).</td>
<td>If students give an incorrect answer, see Teacher Note 3.</td>
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<tr>
<td>4. What part of the brownies was left in the second pan?</td>
<td>Expect to hear ( \frac{2}{5} ).</td>
<td>If students give an incorrect answer, see Teacher Note 3.</td>
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<td>The teacher says or does...</td>
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<td><strong>5.</strong> Max decided to place all of the brownies in the same pan. How can we count what part of a pan of brownies are left? Some of the brownies are not the same size. That makes them hard to count. How can we make them the same size? What is the least common multiple of 3 and 5?</td>
<td>Expect to hear 15.</td>
<td>Tell students to count by 3’s, writing down the numbers as they say them. Repeat, counting by 5’s. Ask “What is the first number that is in both lists?” Mathematicians call that number the <strong>least common multiple</strong>.”</td>
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<tr>
<td><strong>6.</strong> How do you know?</td>
<td>Expect to hear $3 \cdot 5 = 15$.</td>
<td>Explain that $\frac{5}{5} = 1$. So $\frac{1}{3} = \frac{1 \cdot 5}{3 \cdot 5} = \frac{5}{15}$. Similarly, lead students to find $\frac{2}{5} = \frac{6}{15}$. (If students require additional instruction for this step, refer to the lesson <strong>Equivalent Fractions</strong>.)</td>
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<tr>
<td><strong>7.</strong> Let’s find a fraction equivalent to $\frac{1}{3}$ that has a denominator of 15. So $\frac{1}{3} = \frac{5}{15}$. Similarly, lead students to find $\frac{2}{5} = \frac{6}{15}$.</td>
<td>Expect to hear $\frac{5}{15}$.</td>
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<td><strong>8.</strong> Mathematicians say that 15 is the <strong>least common denominator</strong> of $\frac{1}{3}$ and $\frac{2}{5}$.</td>
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<td><strong>9.</strong> Let’s divide the pan into 15 congruent rectangles. Now we can shade in the remaining brownies! On your worksheet, draw Max’s pan of brownies now.</td>
<td>Verify that students have shaded $\frac{11}{15}$.</td>
<td>You may need to discuss with students that an easy way would be to make $3 \times 5$ array. Model the correct drawing with the students. Ask the student(s) to count the number of pieces aloud.</td>
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</table>
The teacher says or does...

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<tr>
<th>10. Let’s write a number sentence that describes Max’s brownies.</th>
<th>Expect students to say or do...</th>
<th>If students do not, then the teacher says or does...</th>
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<tbody>
<tr>
<td>Verify that students write $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15} = \frac{11}{15}$.</td>
<td>Model for students.</td>
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<td>11. How are the numbers 3, 5, and 15 related? (Emphasize that 15 is the least common multiple of 3 and 5 and how students know that.)</td>
<td>15 is the least common multiple of 3 and 5 or $3 \cdot 5 = 15$.</td>
<td>Refer to the skip counting lists in Step 5.</td>
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<td>12. Let’s write down how we solved this problem. That will help us solve other problems in the future.</td>
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<td>13. Continue working a variety of problems. Include problems that have like denominators.</td>
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**Teacher Notes:**

1. When using diagrams, emphasize counting the number of parts in the whole (the denominator) and the number of parts shaded (the numerator). Relate the number of parts in the wholes with the least common multiple. Stress the relationships between the drawings and the number sentences.

2. This lesson emphasizes that addition requires quantities to be the same size ($15^{th}$s in the case of the student probe) and that we are counting the total number of like-sized pieces. Students should be encouraged to use diagrams whenever needed.

3. Say: “How many brownies fill the pan?” Expect to hear 3. Say: “How many brownies were left?” Expect to hear 1. Say: “We call this $\frac{1}{3}$ of a pan. The denominator tells us how many brownies the pan will hold. The numerator tells us how many brownies are left in the pan.” Repeat, if necessary for $\frac{2}{5}$. If students are unable to name the fractions, refer to the lesson Naming Fractions.

4. Finding the least common multiple of the denominators is the most efficient way to solve these addition problems. However, any common multiple will suffice. For example, for the problem $\frac{1}{4} + \frac{1}{6}$ the least common multiple of 4 and 6 is 12, but many students will simply multiply and obtain 24. In this case the answer will be $\frac{10}{24}$ which simplifies to $\frac{5}{12}$. 


Variations

1. Suggested additional introductory problem: Marie’s Brownies
   - Say: “Marie made two pans of brownies, too. Her pans were also the same size. Marie cut one pan into 4 brownies of the same size and the other pan into 6 brownies of the same size. On your worksheet draw how the brownies were cut.”
   - Verify that the students divide each rectangle correctly.
   - Say: “After the party, there was one brownie left in the pan that had been cut into 4 brownies and one brownie left in the pan that had been cut into 6 brownies. Now shade the brownies that are left in each pan.”
   - Verify that students shade 1 brownie in the first pan and 1 brownie in the second pan.
   - Say: “What part of the first pan is shaded?” Expect to hear \( \frac{1}{4} \). Say: “What part of the second pan is shaded?” Expect to hear \( \frac{1}{6} \).
   - Say: “Marie decides to put all of the remaining brownies into the same pan. How can we count what part of a pan of brownies are left? Some of the brownies are not the same size. The brownies must be the same size for us to count them. How can we make them the same size?”
   - Say: “What is the least common multiple of 4 and 6?” Expect to hear 12. Say: “How do you know?” Expect to hear because 12 is the smallest number that is divisible by 4 and by 6. (Some students will say 24 because \( 4 \cdot 6 = 24 \). See the “Teacher Notes” section to address that response.) Say: “Let’s find a fraction equivalent to \( \frac{1}{4} \) that has a denominator of 12.” Expect to hear \( \frac{3}{12} \). Similarly, lead students to find \( \frac{1}{6} = \frac{2}{12} \).
   - Say: “Let’s divide the pan into 12 congruent rectangles.” (You may need to discuss with students that an easy way would be to make an array.) Say: “Now we can shade in the remaining brownies!” Verify that students have shaded \( \frac{5}{12} \).
   - Say: “Let’s write the number sentences that explain what we did.”

2. Use a variety of shapes for the wholes, such as circles, representing pies or pizzas.
3. Cuisenaire Rods are a good model of fraction addition for students who would benefit from a hands-on representation.
4. Once students become comfortable with sums less than 1, problems with sums greater than 1 may be introduced. For example, \( \frac{3}{4} + \frac{2}{3} = \frac{17}{12} = \frac{5}{12} \).

Formative Assessment

Find \( \frac{3}{8} + \frac{1}{6} \). Write a number sentence that explains what you did. You may use the rectangles to help you find the answer.
References


Combining Brownies

Max's Brownies

The least common multiple of 3 and 5 is ________________, because ________________

______________________________.

\[ \frac{1}{3} = \frac{\square}{15} \]

Number Sentence: \[ \frac{1}{3} + \frac{2}{5} = \frac{\square}{15} + \frac{\square}{15} = \frac{\square}{\square} \]

\[ \frac{2}{5} = \frac{\square}{15} \]

Find \( \frac{1}{4} + \frac{1}{6} \).

1. Divide the rectangle into 4 congruent rectangles. Shade \( \frac{1}{4} \) of the rectangle.

2. Divide this rectangle into 6 congruent rectangles. Shade \( \frac{1}{6} \) of the rectangle.
3. What is the LCM of 4 and 6?__________
4. \( \frac{1}{4} = \underline{} \quad \frac{1}{6} = \underline{} \)

5. Divide the rectangle into \__________ \ congruent rectangles. Shade \( \frac{1}{4} + \frac{1}{6} \) of the rectangle.

\( \underline{} \) of \underline{} \ rectangles are shaded.

6. Complete the number sentence:

\( \frac{1}{4} + \frac{1}{6} = \underline{} + \underline{} = \underline{} \)

This is how we solve problems when we want to add fractions with different denominators:

**Step 1**: What is the LCM of 4 and 6?

**Step 2**: Rewrite \( \frac{1}{4} \) using \underline{}.

Rewrite \( \frac{2}{3} \) using \underline{}.

From Step 2

\( \frac{1}{4} + \frac{2}{3} = \underline{} + \underline{} = \underline{} \)
Add the following fractions.

1. \( \frac{1}{2} + \frac{1}{5} \)

2. \( \frac{3}{4} + \frac{1}{8} \)

3. \( \frac{2}{5} + \frac{3}{8} \)

4. \( \frac{1}{3} + \frac{4}{7} \)

5. \( \frac{3}{10} + \frac{1}{12} \)
Marie’s Brownies (Optional)

The least common multiple of 4 and 6 is ________________, because ________________

_______________________________.

\[ \frac{1}{4} = \square \]

Number Sentence: \[ \frac{1}{4} + \frac{1}{6} = \square + \square = \square \]

\[ \frac{1}{6} = \square \]